



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

TANGENTS TOUCHING A SURFACE IN TWO POINTS.

By PROF. F. H. LOUD, Colorado Springs, Colo.

Professor Boyd, in his "Simple Proof of a Theorem with reference to Tangents Touching a Surface in Two Points" (p. 109 of the current volume of the *ANNALS*), has either adopted a different nomenclature from Salmon's or has inadvertently written "triple point" for "double."

A line, tangent to the surface at P , meets it again in $n - 2$ points, and as all these are points of the curve Q , which is of the n th order, it is obvious that Q is met at P , by the *general* line in the tangent plane, at only two points; hence P is a *double* point. The tangents to a plane curve from a *double* point are less by six than from a point of the plane to a non-singular curve of the same order (Salmon, *Higher Plane Curves*, § 79); hence Professor Boyd's conclusion follows.

HILL'S THEORY OF JUPITER AND SATURN.*

The theory of the motions of Jupiter and Saturn is one of the most complicated and difficult which our solar system presents. This theory was for many years an unsolved question with the mathematicians of the last century. The observations showed inequalities in the motions of these planets that could not be accounted for by the theories then existing. Euler took up this question in 1748, and it was the subject of many memoirs by Euler, Lagrange, Laplace, and others. Various hypotheses were introduced, such as a resisting medium in space, and a finite time for the action of the attracting forces, but none of these were satisfactory. Indirectly, however, these investigations were of great value, since the beautiful method of the variation of constants was begun by Euler and completed by Lagrange, and the theory of the secular perturbations of the planets was worked out by Lagrange. The question was solved by Laplace in 1786, by his discovery of the great inequality between these planets arising from the small divisor introduced by integration, and depending on the relation that five times the mean motion of Saturn is nearly equal to twice the mean motion of Jupiter. Although the numerator of this coefficient is of the third order of the eccentricities, this inequality amounts to 20' in the longitude of Jupiter, and to 48' in that of Saturn.

In the present work Mr. Hill has made a very complete investigation of all the perturbations of these planets, both those produced by their mutual

* A New Theory of Jupiter and Saturn. By G. W. Hill. (*Astronomical Papers of the American Ephemeris and Nautical Almanac*).

action and those caused by the other planets. The method employed is that of Hansen (*Auseinandersetzung einer zweckmässigen Methode, etc.*), except that Mr. Hill has taken the time as the independent variable. The work is divided into thirty chapters, filling 576 quarto pages. The table of contents gives a good idea of the thoroughness with which the numerical calculations have been made. The disturbing forces have been taken into account to the third order, and in the final results the coefficients are carried out to the third decimal. In Chap. XXVIII an interesting comparison is made of the theories obtained with some preliminary normal positions of the planets, and corrections are derived to the elements assumed for computing the perturbations. After introducing a new mass of Uranus, and correcting the elements and perturbations, the residuals are as follows :—

Saturn.

1753	$\Delta\lambda = + 1''.06$	1844	$\Delta\lambda = + 0''.34$
1757	$- 0.81$	1851	$+ 0.31$
1761	$+ 0.02$	1858	$- 0.38$
1811	$- 0.44$	1866	$- 0.17$
1822	$- 0.26$	1874	$- 0.41$
1837	$+ 0.20$	1882	$+ 0.53$

Jupiter.

1757	$\Delta\lambda = + 0''.35$	1867	$\Delta\lambda = + 0''.94$
1759	$+ 0.05$	1870	$- 0.10$
1819	$- 1.36$	1874	$- 0.67$
1855	$+ 0.65$	1877	$- 0.48$
1858	$+ 0.12$	1878	$+ 0.05$
1861	$+ 0.28$	1880	$- 0.39$
1864	$+ 0.53$		

This is an excellent agreement, and it shows that the new elements of the planets, and the perturbations derived from them, are very nearly correct. The coefficients of the great inequalities are :

Jupiter.

1196''.14

Saturn.

2907''.86

It remains for Mr. Hill to compare his theories with all the observations made since 1750, to make the final corrections to his elements and perturbations, and to form tables for the motions of the planets. When completed in the thorough manner in which it has been carried on this work will form, we think, the greatest contribution ever made by our country to theoretical astronomy.